Naval Research Laboratory

Washington, DC 20375-5000

NRL Memorandum Report 6831

Recent Advances in Free Electron Laser Theory

C. M. Tang

Beam Physics Branch
Plasma Physics Division

June 11, 1991

Approved for public release; distribution unlimited.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
1991 June 11

3. REPORT TYPE AND DATES COVERED
Interim

4. TITLE AND SUBTITLE
Recent Advances in Free Electron Laser Theory

5. FUNDING NUMBERS
JO# 47-0899-0-1

6. AUTHOR(S)
C. M. Tang

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NRL
Washington, DC 20375-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL Memorandum Report 6831

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
ONR
Arlington, VA 22217

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Free electron lasers (FELs) have advanced to the stage where the experiments may be considered to have verified the essential FEL theories. The future research in FELs will be strongly tied to applications. There exist many types of lasers and optical techniques to obtain the required laser characteristics in the optical regime. FELs can make significant impact in the IR, UV and x-ray regimes, where conventional lasers are limited. Competition with other sources also requires FELs to be compact and user oriented. The theory developments addressing some of these issues will be presented in this paper.

14. SUBJECT TERMS
Free electron lasers
Ultra-violet radiation (UV) and x-ray

15. NUMBER OF PAGES
22

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

CONTENTS

RECENT ADVANCES IN FREE ELECTRON LASER THEORY

1. Introduction

Free electron lasers comprise a class of potentially efficient devices capable of generating high quality coherent radiation, continuously tunable from the microwave region to the x–ray, at high average and/or high peak powers. The basic mechanisms of FEL physics have been verified. The future trends for FELs are: i) to extend the frontier of FELs into longer (IR) and shorter (UV and x–ray) wavelength regimes where conventional sources are limited, ii) to provide user oriented FELs at user facilities and iii) eventually to make compact, inexpensive, commercially affordable and easy to use FELs.

The proof–of–principle IR FELs are easier to make than FELs at shorter wavelength, because the electron beam requirement is less stringent. To make IR FELs into a compact and practical device is another matter. Common FEL physics issues are beam quality requirements, radiation guiding and micro–wigglers. Additional issues of importance to the IR are: i) radiation sidebands (physics related to potential high power operation), ii) use of waveguides (physics related to diffraction and group velocity effects) and iii) pulse slippage (physics related to short electron beams from rf driven accelerators).

Generation of FEL radiation in the UV and x–ray regime is challenging. Beam quality requirements become very stringent. To reduce diffraction losses, radiation guiding is necessary. If conventional wigglers are to be used, it might require a large number of periods. The effect of wiggler field errors on the FELs becomes important. To reduce the overall size, micro–wigglers or electromagnetic wigglers, and radiation at harmonics are topics to be examined.

In this paper, we will concentrate on topics of importance for generation of radiation in the UV and x–ray regimes. The advances in the theory and configurations will be highlighted.

2. Effect of Beam Quality on Radiation Guiding

All the experiments have demonstrated the importance of high quality electron beams. It is desirable to focus the electron beam and to obtain high electron beam density with small axial velocity spread in the FEL interaction region. At large electron beam density, FELs can operate in the radiation "guided" regime.[1–12] Optical guiding overcomes diffraction losses and improves the growth rate and the efficiency.

Manuscript approved April 16, 1991.

A. Radiation Guiding

The FEL mechanism is most effective when the radiation beam just overlaps the electron beam, such that the filling factor $f = \sigma_b/\sigma_r$ is approximately unity, where σ_b is the area of the electron beam and σ_r is the area of the radiation. For many experiments it is desirable to have the wiggler length be much longer than the Rayleigh length. If the radiation cannot be confined or guided by a waveguide structure, it is important to overcome free-space diffraction by utilizing an important property of the FEL – radiation guiding.

To illustrate the concept of radiation self–focusing in the high gain regime, consider a radiation field with a vector potential given by

$$\mathbf{A}_r(r,z,t) = -A(r,z)\exp[i\omega(z/c-t)]\hat{\mathbf{e}}_x/2 + c.c., \tag{1}$$

where $A(r,z) = |A(r,z)|e^{i\phi(r,z)}$ is the axially symmetric complex amplitude of the radiation field, ω is the frequency, and c.c. denotes the complex conjugate. The wiggler is linearly polarized with flat pole faces, where A_w is the peak vector potential, k_w is the wavenumber and L_w is the length of the wiggler.

The wave equation is

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)a(r,z) = \frac{\omega^2}{c^2}[1-n^2(r,z,a)]a(r,z), \tag{2}$$

where

$$n(r,z,a) = 1 + \frac{1}{2}\frac{\omega_b^2(r,z)}{\omega^2}\left\langle\frac{e^{-i\psi}}{\gamma_0}\right\rangle\frac{F_1 K}{|a(r,z)|} \tag{3}$$

is the index of refraction associated with the medium and is, in general, complex and a nonlinear function of $a(r,z)$. Here, $\omega_b(r,z) = [4\pi|e|^2 n_b(r,z)/m_0]^{1/2}$ is the plasma frequency associated with the electron beam density $n_b(r,z)$, $a(r,z) = |e|A/\sqrt{2}m_0c^2$ is the normalized radiation vector potential, $K = (|e|/m_0c^2)(A_w/\sqrt{2})$ is the normalized wiggler vector potential and $F_1 = J_0(b) - J_1(b)$, $b = K^2/2(1+K^2)$. The $\langle\rangle$ denote an ensemble average over all electrons within a ponderomotive wavelength. The imaginary part of n and the real part of n represent the gain and the refractive properties of the radiation, respectively. Gain and refraction are coupled in FELs.

The radiation field $a(r,z)$ can be represented in terms of a complete set of basis functions[5-9]

$$a(r,z) = \sum_m a_m(z) L_m\left(\frac{2r^2}{r_s(z)^2}\right) \exp\left[-[1 - i\alpha(z)]\frac{r^2}{r_s^2(z)}\right], \tag{4}$$

where $m = 0, 1, 2, \cdots$. In Eq. (4), $a_m(z)$ are the complex amplitude coefficients, $r_s(z)$ is the radiation spot size, $\alpha(z)$ is related to the radius of curvature of the radiation beam wavefront, $R = -(\omega/2c)r_s^2/\alpha$ is the radius of curvature, and L_m is the Laguerre polynomial.

Let us assume that the radiation beam is approximately Gaussian and the a_0 mode is a good approximation to the radiation field. The unknown quantities a_m, r_s, and α can be solved for self-consistently.

The beam is called perfectly guided when self-similar solutions exist, i.e., the spot size and the curvature remain constant. The complex amplitude of the radiation grows exponentially,

$$a(r,z) = a_0(z=0)\exp[(\Gamma - i\Delta k)z]\exp\left[-r^2\left(\frac{1}{r_s^2} + i\frac{k}{2R}\right)\right], \tag{5}$$

where Γ is the growth rate and Δk is the phase shift. Guided radiation beams can be obtained in the small signal, exponential gain regime for a uniform electron beam radius.

Solutions of r_s, R, Γ, and Δk are given in Table I[9] for parabolic electron beams, where $n_b(r) = n_0(1 - r^2/r_b^2)$, and n_0 is the electron beam density on axis. The filling factor is defined as

$$f = \sigma_b/\sigma_r, \tag{6}$$

where $\sigma_b = \pi r_b^2/2$, and $\sigma_r = \pi r_s^2$. For large filling factors ($f > 1$), the scaling relations reduce to those of the 1-D limit.

Optical guiding is important when the interaction length is much longer than the Rayleigh length, and the Rayleigh length is comparable to the e-folding length, i.e.,

$$L_w >> z_R > 1/\Gamma, \tag{7}$$

where $z_R = \pi r_s^2/\lambda$ is the Rayleigh length associated with the radiation spot size and λ is the radiation wavelength. Since it is desirable to make the FEL compact, the FEL wiggler length can be reduced by using a shorter wiggler wavelength, which leads to the requirement of a small electron beam area σ_b.

In addition to the Gaussian radiation beam approximation, the derivation of the radiation guiding presented above assumed a cold beam with a given electron beam radius and no betatron oscillation effect.

B. Beam Quality

The effective energy spread of the beam does not degrade the performance of the FEL in the high gain Compton regime if the variation in the phase of electrons with different axial velocities is small over the period of ponderomotive potential at the end of an e–folding length. The effective energy spread criterion can be written as

$$(k_w + k)\Delta v_z L_e / v_{z0} << \pi/2, \tag{8}$$

where Δv_z is the axial velocity spread, $L_e = 1/\Gamma$ is the e–folding length. A spread in v_z is related to a spread in γ_z by $\Delta\gamma_z\gamma_{z0} = \gamma_{z0}^2 \Delta v_z / c$, where $\gamma_0 = (1 + K^2)^{1/2}\gamma_{z0}$ and $\gamma_z = \gamma_{z0} + \Delta\gamma_z = \left[1 - (v_{z0} + \Delta v_z)^2/c^2\right]^{-1/2}$. The energy spread limit can be written as

$$\frac{\Delta\gamma/\gamma_0}{\Gamma/k_w} < \frac{1}{2}. \tag{9}$$

Transverse emittance, ϵ, is an important measure of beam quality. We define $\pi\epsilon$ to be the area of the smallest ellipse in $x, x' = dx/dz$ transverse phase space which encloses the particles in the beam. We will assume that the beam is symmetric and $\epsilon = \epsilon_x = \epsilon_y$. In some accelerators the emittances in the x and in the y planes are different. We will consider the "envelope," or "edge," emittance of the beam

$$\epsilon_x = 4\left[\langle x^2\rangle\langle (x')^2\rangle - (\langle xx'\rangle)^2\right]^{1/2} = 4\epsilon_{x,rms}, \tag{10}$$

where $\epsilon_{x,rms}$ is the rms emittance. As energy increases, ϵ decreases; but the normalized emittance

$$\epsilon_n = \beta\gamma\epsilon \tag{11}$$

is invariant through the entire linac and beam transport system for linear focusing systems.

Emittance produces an effective axial velocity spread

$$\frac{\Delta v_{z,\epsilon}}{c} = \frac{1}{2}\frac{\epsilon^2}{r_b^2}. \tag{12}$$

Substitute Eq.(12) into(8), and the criteria on emittance is

$$\epsilon^2 < \beta_{z0}\frac{\pi r_b^2}{kL_e}, \tag{13}$$

where $\beta_{z0} = v_{z0}/c$. For electron beams properly matched into the wiggler, i.e., $r_b^2 = \epsilon/K_\beta$, and $\beta_{z0} \simeq 1$,

$$\epsilon << \frac{\lambda}{2}\frac{1}{K_\beta L_e},$$

where $K_\beta = Kk_w/\gamma_0$ is the betatron wavenumber. The emittance criteria above can be interpreted in terms of radiation guiding. For perfectly guided radiation ($f \simeq 1$), the Rayleigh length, $z_R \simeq \pi r_b^2/2\lambda$, is approximately the e–folding length. To obtain ($f \simeq 1$), the approximate criterion for the emittance is

$$\frac{\pi\epsilon}{\lambda} \lesssim 1. \tag{14}$$

C. The Effect of Beam Quality on Radiation Guiding

In a recent paper, Ref. 11, the growth rate is derived including simultaneously the effects of: i) energy spread $\Delta\gamma/\gamma_0$, ii) rms emittance ϵ_{rms}, iii) betatron oscillations and focusing of the electron beam, iv) diffraction and guiding and v) frequency detuning.

Reference 11 assumed a radiation field,

$$E(r,\omega)e^{-i\mu k_w z}e^{-i\omega(t-z/c)}\hat{e} + c.c., \tag{15}$$

where

$$E(r) = \begin{cases} e^{-\chi r^2/2r_b^2}, r \leq r_b; \\ AH_0^{(1)}(r\sqrt{\mu}), r \geq r_b, \end{cases} \tag{16}$$

where $H_0^{(1)}$ is the Hankel function, and r_b is the edge of the electron beam radius. The expression for $E(r)$ is exact for $r \geq r_b$ and approximate for $r \leq r_b$. Additional assumptions are that the electron beam's energy distribution is Gaussian. There is weak focusing due to the wiggler in both x and y so that the electrons execute betatron oscillations in both x– and y–directions and the beam has an axially symmetric and uniform "water-bag" distribution in transverse phase space.

From the dispersion relation obtained from Vlasov–Maxwell equations, the imaginary part of μ gives the growth rates and it can be written in the following form

$$\Gamma = \tilde{\Gamma} G \left[\quad \frac{4\pi\epsilon_{rms}}{\lambda_r}, \quad \frac{\Delta\gamma/\gamma}{\tilde{\Gamma}/k_w}, \quad \frac{K_\beta}{\tilde{\Gamma}}, \quad \frac{(\omega - \omega_r)/\omega_r}{\tilde{\Gamma}/k_w} \quad \right], \tag{17}$$

where ω_r and λ_r are the resonant frequency and wavelength, $\tilde{\Gamma} = 4(|e|/m_o c^2)^{1/2} F_1 k_w (K/(1+K^2)^{1/2})(I_b/\gamma_0)^{1/2}$ and I_b is the peak current. The intrinsic efficiency can also be obtained from the same formalism, by plotting the real part of the variable μ.

Figures 1a,b are plots of the growth rates as a function of emittance for different values of the betatron wavenumber strength at the resonant frequency for (Fig. 1a) no energy spread $(\Delta\gamma/\gamma_0)/(\tilde{\Gamma}/k_w) = 0$ and (Fig. 1b) a small energy spread $(\Delta\gamma/\gamma_0)/(\tilde{\Gamma}/k_w) = 0.2$. The growth rate $\tilde{\Gamma} \simeq \Gamma_0$, can be interpreted as the growth rate of the "guided" radiation when ($f = 1$), (See Table I). Note that for $4\pi\epsilon_{rms}/\lambda_r > 1$ the growth rate in Figs. 1a,b are significantly degraded, consistent with Eq. (14). Also note that for $(\Delta\gamma/\gamma_0)/(\tilde{\Gamma}/k_w) = 0.2$, all the growth rate curves have decreased, consistent with the simple minded condition Eq. (9). Comparison with 3-D self-consistent FEL simulation results, by J. Goldstein with FELEX code and J. Wurtele with TDA code, at maximum growth rates at the parameter regimes shown in Figs. 1a,b are within 5%. These universal curves provide the quantitative reduction of the growth rates due to energy spread and emittance.

The limitations of this calculation become relevant when the radiation field is significantly different from the form shown in Eq. (16). This can happen in a variety of situations. The agreement between simulation and the dispersion relation (17) is degraded as the frequency moves off resonance due to the presence of higher order modes. The agreement will probably be poor, in general, for filling factor $f >> 1$ and for $f << 1$. The calculation is derived for the growth rate of the fastest growing mode. If this mode does not dominate over other modes, many modes coexist and the simulation deviates from this calculation. In addition, the distribution function of the electron beam may be different from that used in the derivation. The effect of the distribution function on the scaling may be important, based on conclusions from 1–D simulations[13] and 1–D Vlasov theory.[14]

3. Micro–Wigglers and Harmonics

Requirements for compact FELs for all radiation wavelengths are short period wigglers, low energy electron beams and/or operation in the harmonics. The wavelength of the radiation can be written as

$$\lambda = \frac{\lambda_w}{n\beta_{z0}} \frac{1+K^2}{(1+\beta_{z0})\gamma_0^2}, \tag{18}$$

where n is the harmonic number, λ_w is the wiggler wavelength, $\beta_{z0} = (1 - 1/\gamma_{z0}^2)^{1/2}$ and $\gamma_{z0} = \gamma_0/(1+K^2)^{1/2}$.

For a cold electron beam with very small emittance, the ideal expression for gain[9] in the low gain regime can be written as

$$G = \frac{\pi^2}{\sigma_r} \frac{|e|}{m_o c^3} \frac{I_b}{\gamma_0} \left(\frac{K\lambda_w}{\gamma_{z0}}\right)^2 N_w^3 Q(n,K), \tag{19}$$

$$Q(n,K) = \frac{n^2}{1+K^2} F_n^2, \tag{20}$$

$$F_n = J_{(n-1)/2}(b_n) - J_{(n+1)/2}(b_n) \tag{21}$$

where N_w is the number of wiggler periods and $b_n = n\xi$ and $\xi = K^2/2(1+K^2)$. Harmonic operation requires the parameter K to be larger than 1. This requirement becomes clear by the plot of the Q–factor in Fig. 2.[15]

To generate large values of K at short wavelength is difficult. Reference 15 outlines a method of obtaining K values from 1 to 4 for micro–wigglers with period $\lambda_w \sim 1$ to 5 mm, by pulsing electromagnetic wigglers. The energy spread and emittance requirement can be minimized by operating with wigglers with a small number of periods in an oscillator. Methods of suppressing unwanted harmonics are proposed in Ref. 16.

4. Laser Pumped Harmonic FELs in the XUV Regime

Another method of generating short period wigglers is to use intense laser pulses. This concept had not been very promising because of the low laser intensities and the stringent beam requirements. Recent advances in laser technology have demonstrated very high intensities. For example, intensities greater than 10^{18} W/cm^2 for Nd:glass YAG have been demonstrated. These intense laser pulses can generate the normalized vector potential $K = (|e|/m_0c^2)(\lambda_L E_L/2\sqrt{2}\pi)$ of up to about 3, where λ_L is the wavelength of the pump laser, $E_L = 9.15 \times 10^{-2}\sqrt{I}$ is the peak electric field of the linearly polarized pump laser in sv/cm and I is the laser intensity in W/cm^2.

A schematic of a laser pumped FEL[17] is shown in Fig. 3, where the wavelength of the coherent backscattered radiation is

$$\lambda_n = \frac{1+K^2}{(1+\beta_0)^2\gamma_0^2}\frac{\lambda_L}{n}, \tag{22}$$

where n is the harmonic number and $\beta_0 = (1 - 1/\gamma_0^2)^{1/2}$. For the Nd:glass YAG laser at wavelength of 1 μm and intensity of 10^{19} W/cm^2, $K = 1.9$. For an electron beam of 2 MeV ($\gamma_0 = 5$), the fundamental FEL wavelength is $\sim$ 460 Å and the 5th harmonic wavelength is $\sim$ 90 Å.

The 1-D growth rate[17] of the harmonic is

$$\Gamma_{L,n} = 2(1+\beta_0)^{-2/3}\Gamma_n, \tag{23}$$

where

$$\Gamma_n = \sqrt{3}\left(f\frac{\pi}{2}\frac{nF_n^2}{\lambda_L}\frac{|e|}{m_oc^3}\frac{I_b}{\gamma_0}\right)^{1/3}\left(\frac{K}{\gamma_0 r_b}\right)^{2/3} \tag{24}$$

is the 1-D growth rate[9,18] of the harmonic in the linearly polarized static wiggler of period λ_L written in terms of the beam current I_b, where $f = \sigma_b/\sigma_n$ is the filling factor, σ_b is the cross sectional area of the electron beam, σ_n is the cross sectional area of the harmonic radiation and r_b is the radius of the electron beam. The growth rate of the laser pumped FEL in terms of electron density is

$$\Gamma_{L,n} = 2\sqrt{3}\pi\left(\frac{f(1+K^2)}{(1+\beta_0)^2\gamma_0^3}\frac{b_nF_n^2}{\lambda_p^2\lambda_L}\right)^{1/3}, \tag{25}$$

where $\lambda_p = 2\pi/k_p$ is the wavelength of the plasma period, $k_p = (4\pi|e|^2n_p/m_o)^{1/2}/c$ is the plasma wavenumber associated with the electron beam and n_p is the electron density in the beam. The derivation of the growth rate expression assumed the electron beam is in the single particle regime, i.e., $\lambda_p >> \lambda_L$. Expressions for growth rates in the radiation guided regimes were also obtained.[17]

The feasibility of the laser pumped FEL operating in the high gain Compton regime requires very high beam density and very good beam quality. The necessary electron beams do not exist at the present time, but many cathode programs under development are working to approach that goal.

5. Laser Pumped Plasma FEL in the XUV and X–Ray Regime

A novel method of generation of the stimulated harmonic radiation from dense stationary plasmas, rather than electron beams, has been proposed.[19] The mechanism requires collective regime operation, i.e., $\lambda_p << \lambda_L$. The schematic configuration of the laser pumped plasma FEL is shown in Fig. 4. The intense input laser at wavelength λ_L enters the plasma and causes the electrons in the plasma to oscillate in the transverse and axial directions. As a consequence of the collective effects, the energy associated with the axial electron motion in the plasma obeys

$$\gamma_0(1+\beta_0) = \sqrt{1+K^2}. \tag{26}$$

The resonant radiation propagates in the opposite direction of the input laser, similar to the laser pumped FEL. The wavelength of the stimulated radiation for the nth harmonic is

$$\lambda_n = \lambda_L/n. \tag{27}$$

The advantages of this configuration for generation of stimulated radiation are the high growth rates associated with the high electron density in the plasma and the elimination of an electron beam source. Continuous tuning of the radiation will depend on the tunability of the pump laser.

The growth rate is

$$\Gamma_{L,n} = \sqrt{3}\pi\left(\frac{2b_n F_n^2}{\lambda_p^2\lambda_L}\right)^{1/3}, \tag{28}$$

where λ_p is the plasma wavelength and the filling factor is $f=1$. For Eq. (28) to be valid, it is necessary that $\lambda_L/\lambda_p << 1$.

Similar to all FEL configurations, the axial velocity spread has to be small. In this case,

$$\frac{\Delta v_z}{c} < \frac{Im(\Delta k)}{k_n}, \tag{29}$$

where $Im(\Delta k)$, related to the efficiency, is to be determined from the dispersion relation and $k_n = 2\pi/\lambda_n$. Laser photo-ionization[20-21] may be able to produce the necessary longitudinally cold plasmas.

Take the Nd:glass YAG laser again as the pump laser, and the $11th$ harmonic, the stimulated harmonic wavelength is 910 Å. For a laser intensity of 10^{19} W/cm^2, $K = 1.9$. For a plasma with density of 10^{16}, the plasma wavelength is 3.34×10^{-2} cm. The Bessel function coefficient related to the harmonic number 11 is $(b_{11} F_{11}^2)^{1/3} \simeq 0.55$. The e-folding length is only 16 μm.

6. Summary

Generation of tunable, coherent XUV and x-rays remains a challenge. Some of the methods outlined above are very promising. Extensive theoretical and experimental research in this area is warranted. The electron density and the axial velocity spread of the electron source are the primary limiting factors for generating shorter wavelength radiation via the FEL mechanism.

Acknowledgment

This work is supported by the Office of Naval Research. I would like to thank P. Sprangle, B. Hafizi, E. Esarey, A. Ting, Y. H. Seo, L. H. Yu and R. Warren for valuable discussions.

References

[1] P. Sprangle and C. M. Tang, Appl. Phys. Lett. **39**, 677 (1981); P. Sprangle and C. M. Tang, AIAA J. **19**, 1164 (1981); C. M. Tang and P. Sprangle, *Free-Electron Generators of Coherent Radiation*, Physics of Quantum Electronics, edited by S. F. Jacobs, G. T. Moore, H. S. Pilloff, M. Sargent III, M. O. Scully and R. Spitzer (Addison-Wesley, Reading, MA, 1982), p. 627.

[2] G. T. Moore, Opt. Comm. **52**, 46(1984); G. T. Moore, Ibid. **54**, 121 (1985); G. T. Moore, Nucl. Instrum. Methods in Phys. Research **239**, 19 (1985).

[3] E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele, Phys. Rev. Lett. **54**, 1925 (1985).

[4] M. Xie and D. A. G. Deacon, Nucl. Instrum. Methods in Phys. Research **A250**, 426 (1986).

[5] P. Sprangle, A. Ting, and C. M. Tang, Phys. Rev. Lett. **59**, 202 (1987); P. Sprangle, A. Ting, and C. M. Tang, Phys. Rev. **A30**, 2773 (1987).

[6] B. Hafizi, P. Sprangle, and A. Ting, Phys. Rev. **A36**, 1739 (1987).

[7] P. Sprangle, A. Ting, B. Hafizi, and C. M. Tang, Nucl. Instrum. Methods in Phys. Research **A272**, 536. 1988.

[8] C. M. Tang, P. Sprangle, A. Ting, and B. Hafizi, J. Appl. Phys. **66**, 1549 (1989)

[9] C. M. Tang, B. Hafizi, E. Esarey, A. Ting, W. Marable and P. Sprangle, *The Physics of Particle Accelerators*, AIP Proc., in press.

[10] Y. Seo, V. K. Tripathi and C. S. Liu, Phys. Fluids **B1**, 221 (1989).

[11] L. H. Yu, S. Krinsky and R. L. Gluckstern, Phys. Rev. Lett. **64**, 3011 (1990).

[12] L. H. Yu, S. Krinsky and R. L. Gluckstern, these proceedings (12th Int. FEL. conf., Paris, France, 1990) Nucl. Instr. and Meth.

[13] W. B. Colson, J. Blau and J. H. Park private communications.

[14] L. H. Yu, private communications.

[15] R. W. Warren, "Lasing on Higher Harmonics", these proceedings (12th Int. FEL. conf., Paris, France, 1990) Nucl. Instr. and Meth.

[16] R. W. Warren, "Compound Wigglers Designed to Lase on a Selected Harmonic", ibid.

[17] P. Sprangle and B. Hafizi, NRL Memo. Report No. 6375.

[18] R. C. Davidson, Phys. Fluids **29**, 267 (1986).

[19] P. Sprangle and E. Esarey, NRL Memo. Report No. 6786, and submitting for publications.

[20] P. B. Corkum, N. H. Burnett and F. Brunel, Phys. Rev. Lett. **62**, 1259 (1989).

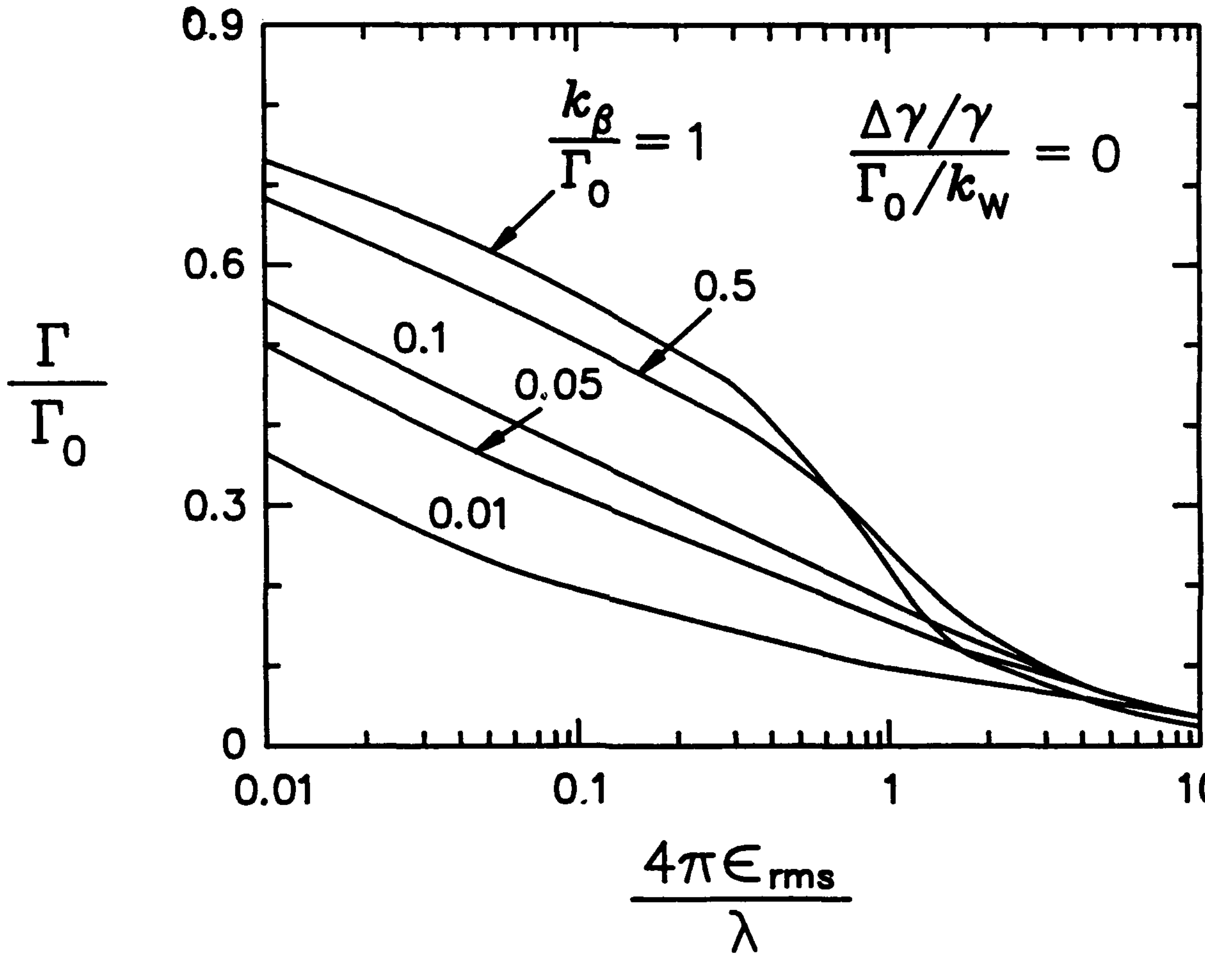
[21] N. A. Burnett and P. B. Corkun, J. Opt. Soc. Am. **B6**, 1195 (1989).

Table I: Guided Radiation for Parabolic Electron Beam

Filling factor, $f = \dfrac{\sigma_b}{\sigma_r}$	$\dfrac{3}{4}\left(\dfrac{3\sqrt{2}k\Gamma_0\sigma_b}{5\pi}\right)^{1/3}$	1
Radiation spot size, r_s	$r_b(2f)^{-1/2}$	$r_b 2^{-1/2}$
Growth rate, Γ	$\dfrac{3}{5}\Gamma_0\dfrac{(6f-1)}{f(4f-1)}$	Γ_0
Radius of curvature, R	$-\dfrac{k\sigma_b}{\pi}\dfrac{(6f-1)^{1/2}}{f(2f-1)^{1/2}}$	$-0.7k\sigma_b$
Phase shift, Δk	$\dfrac{3}{5}\Gamma_0\dfrac{(2f-1)}{f(4f-1)}$	$0.2\Gamma_0$
Intrinsic efficiency, η	$\Delta k/k_w$	$0.2\Gamma_0/k_w$

$$\Gamma_0 = \frac{5}{6}\frac{F_1 K k_w}{(1+K^2)^{1/2}}\left(\frac{\nu}{\gamma_0}\right)^{1/2}$$

$\nu = I_b/17\beta_0$	Budker's parameter
I_b	peak current in kA
$k = 2\gamma_0^2 k_w/(1+K^2)$	radiation wave number
$\sigma_b = \pi r_b^2/2$	area of e-beam
r_b	electron beam radius
$F_1 = J_0(b) - J_1(b)$	$b = K^2/2(1+K^2)$
$K = (\lvert e\rvert/m_0c^2)(A_w/\sqrt{2})$	
$\beta_0 = (1-\gamma_0^{-2})^{1/2}$	



(a)

Fig. 1. Plots of the growth rates as a function for emittance for different values of the betatron wavenumber strength at resonant frequency and a) no energy spread $(\Delta\gamma/\gamma_0)/(\bar{\Gamma}/k_w) = 0$

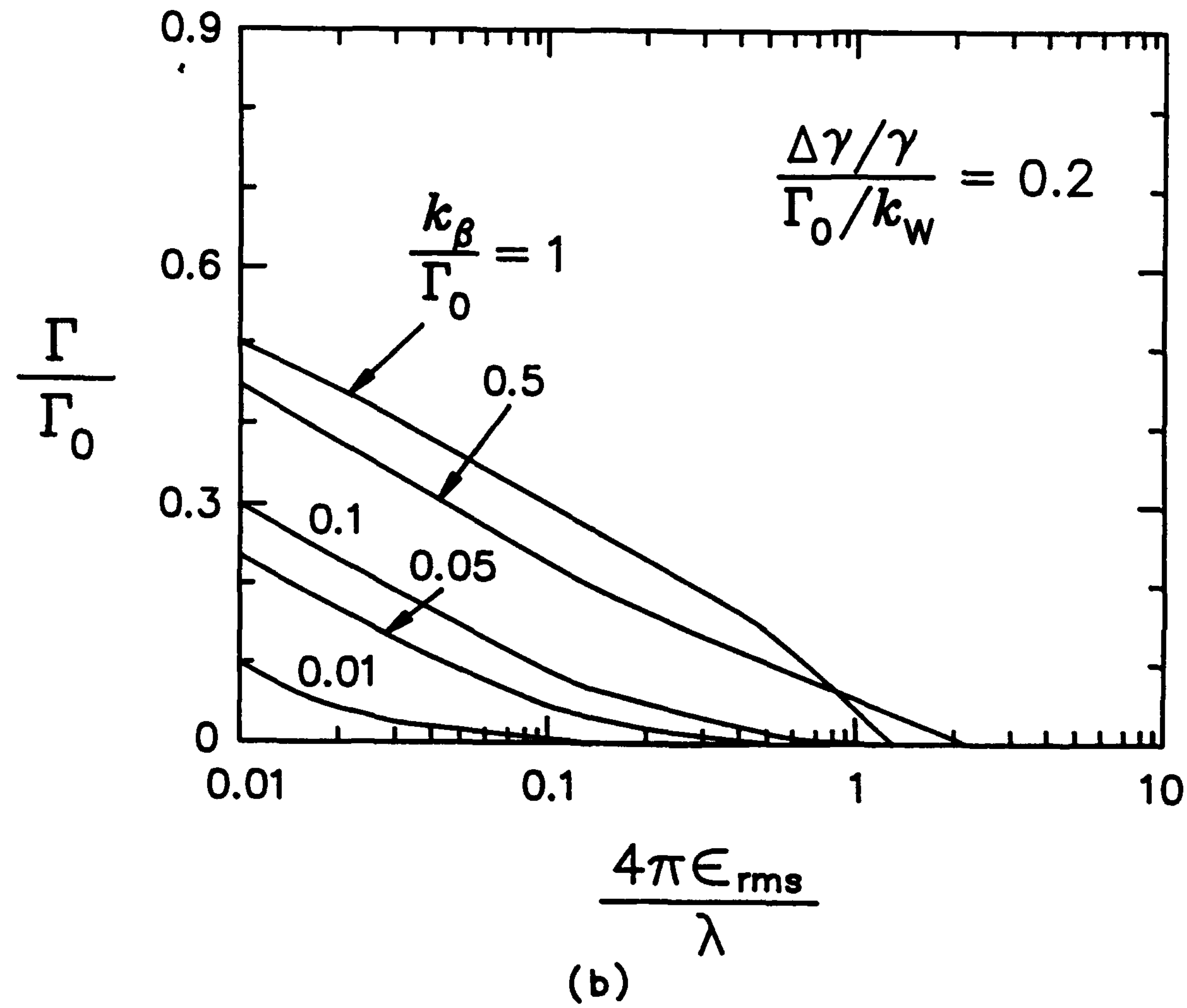
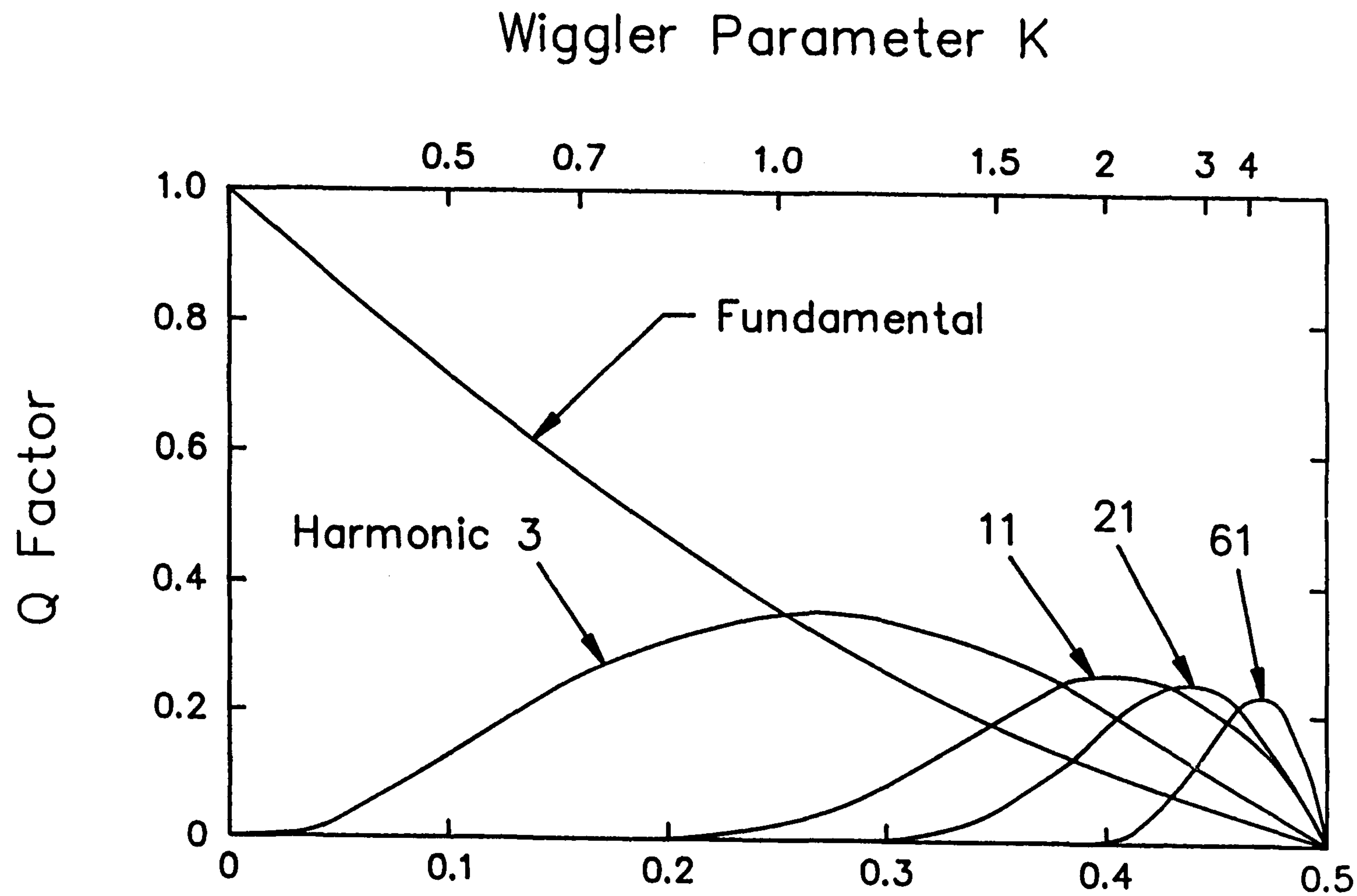


Fig. 1. Plots of the growth rates as a function for emittance for different values of the betatron wavenumber strength at resonant frequency and b) $(\Delta\gamma/\gamma_0)/(\bar{\Gamma}/k_w) = 0.2$.



$$\frac{1}{2}\,\frac{K^2}{1+K^2}$$

Fig. 2 Plots of the Q factor as function of the wiggler parameter K for various harmonic numbers.

Pump Laser Pulse and Electron Pulse Configuration

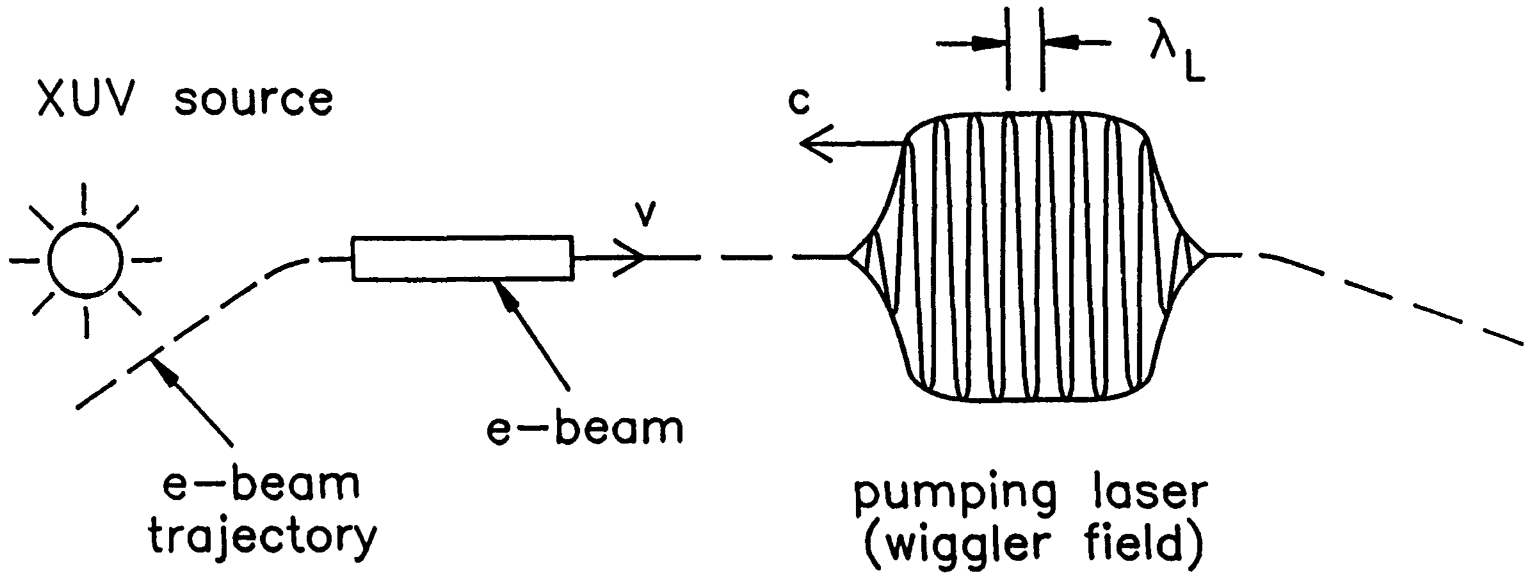


Fig. 3 Schematic of the laser pumped FEL.

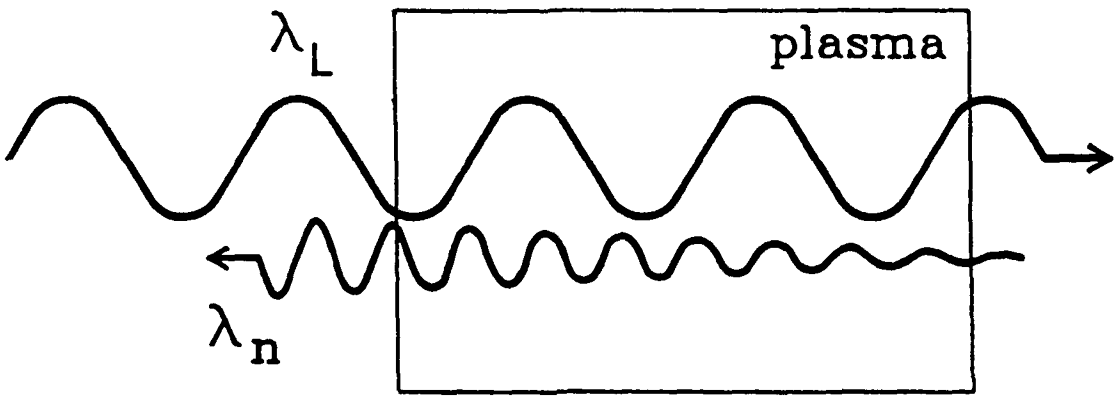


Fig. 4 Schematic of the laser pumped plasma FEL.